Functional renormalization group in the broken symmetry phase: momentum dependence and

two-parameter scaling of the self-energy

Functional renormalization group in the broken symmetry phase: momentum dependence and two-parameter scaling of the self-energy

Andreas Sinner, Nils Hasselmann and Peter Kopietz

Institut für Theoretische Physik, Universität Frankfurt, Max-von-Laue Strasse 1,
60438 Frankfurt, Germany

Abstract
We include spontaneous symmetry breaking in the functional renormalization group equations
for the irreducible vertices of Ginzburg–Landau theories by augmenting these equations by a
flow equation for the order parameter, which is determined from the requirement that at each
renormalization group (RG) step the vertex with one external leg vanishes identically. Using
this strategy, we propose a simple truncation of the coupled RG flow equations for the vertices
in the broken symmetry phase of the Ising universality class in D dimensions. Our truncation
yields the full momentum dependence of the self-energy $\Sigma(\boldsymbol{k})$ and interpolates between
lowest-order perturbation theory at large momenta $\boldsymbol{k}$ and the critical scaling regime for small $\boldsymbol{k}$.
Close to the critical point, our method yields the self-energy in the scaling form
$\Sigma(\boldsymbol{k}) = k_c^2 \sigma^-(|\boldsymbol{k}|\xi, |\boldsymbol{k}|/k_c)$, where ξ is the order parameter correlation length, k_c is the
Ginzburg scale, and $\sigma^-(x, y)$ is a dimensionless two-parameter scaling function for the broken
symmetry phase which we calculate explicitly within our truncation.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

The functional renormalization group (FRG) was invented
by Wegner and Houghton [1] in the early days of the
renormalization group (RG) as a mathematically exact
formulation of the Wilsonian RG. In the past decade this
method has gained new attention. While there are several
equivalent formulations of the FRG involving different types
of generating functionals, in many cases it is advantageous
to formulate the FRG in terms of the generating functional
Γ of the one-particle irreducible vertices, which can be
obtained from the generating functional of the connected Green
functions via a Legendre transformation [2, 3]. Two different
strategies of solving the formally exact FRG equation for
the functional Γ have been developed: the first is based on
the combination of the local potential approximation (LPA)
with the derivative expansion [4, 5]. This approach has
been very successful in obtaining accurate results for critical
exponents [4–7] and is convenient for describing the broken
symmetry phase [8], because it is based on an expansion in

terms of invariant densities, which automatically fulfill all
symmetry requirements.

The other strategy, which was pioneered by Morris [3]
and has been preferentially used in the condensed matter
community to study non-relativistic fermions [9, 10], is based
on the expansion of Γ in powers of the fields, leading to an
infinite hierarchy of coupled integro-differential equations for
the one-particle irreducible vertices. This approach has the
advantage of providing information on the momentum and
frequency dependences of the vertices. However, there have
been only two conceptually different attempts to extend the
hierarchy of FRG flow equations for the vertices arising from
the field expansion into the broken symmetry phase. One
possibility is to include a small symmetry-breaking component
in the initial condition for the self-energy [11–13] and to check
whether this component evolves into a macroscopic value as
the RG flow is integrated. A disadvantage of this scheme is
that the order parameter field and its fluctuations do not appear
explicitly and that one even has to invest some effort to recover
simple mean-field results.

Another possibility for extending the FRG flow equations for the irreducible vertices to the broken symmetry phase was proposed in [14] (see also [15]). The basic idea is to augment the hierarchy of flow equations for the vertices by an additional equation for the flowing order parameter, which is obtained from the requirement that, at each stage of the RG flow, the vertex with one external leg vanishes identically. The purpose of this work is to show how this strategy works in practice. For simplicity, here we shall consider a simple classical scalar φ^4-theory, describing the critical behavior of the Ising universality class; generalizations of our method to quantum systems are straightforward. For example, this method has recently been used to study superconductivity in the attractive electron gas, where the flow equation for the order parameter is equivalent to a generalized BCS (Bardeen–Cooper–Schrieffer) gap equation including fluctuation corrections [16].

Apart from showing how symmetry breaking can be taken into account in the field expansion, in this work we present two additional new results: on the one hand, we propose a simple truncation of the exact hierarchy of the flow equations for the irreducible vertices in the broken symmetry phase which yields the full momentum dependence of the self-energy $\Sigma(\boldsymbol{k})$, interpolating between the perturbative regime for large momenta $k \equiv |\boldsymbol{k}|$ and the critical regime for $k \rightarrow 0$. Different strategies for calculating the $\boldsymbol{k}$-dependence of the self-energy (and, more generally, the momentum dependence of the higher-order vertices) has recently been developed in [17] and in [18]. On the other hand, in this work we present an approximate calculation of the *two-parameter* scaling function $\sigma^-(k\xi, k/k_c)$ describing the scaling of the self-energy of the system slightly below the critical temperature T_c. Here ξ is the order parameter correlation length, and k_c is the Ginzburg scale, which remains finite at the critical point [19]. The corresponding scaling function $\sigma^+(k\xi, k/k_c)$ in the symmetric phase (i.e. for temperature $T > T_c$) has recently been discussed in [20]. The fact that the Ginzburg scale k_c appears in the scaling of thermodynamic variables has been discussed in several recent works [21, 22, 4], but apparently has been ignored in the older RG literature [23–25]. For models with weak interactions, a universal regime, covering complete crossover from the vicinity of the Gaussian fixed point to the vicinity of the Wilson–Fisher-fixed point, exists which can be described completely within a two-parameter scaling theory [21, 22, 4]. For the weakly interacting Bose gas at criticality, the one-parameter scaling function $\sigma_*(k/k_c) = \sigma^-(\infty, k/k_c)$ has been calculated in [26, 27, 17, 18]. However, the full implications of a finite-scale k_c at criticality and the resulting *two-parameter* scaling theory of the correlation function away from criticality has only recently been examined [20]. The field expansion of the FRG allows us to study the complete momentum dependence of correlation functions and is thus ideally suited to exploring the extended universality near the critical temperature T_c.

The structure of the paper is as follows. In section 2 we formulate the exact FRG flow equations for the running order parameter M_Λ and the momentum-dependent self-energy $\Sigma_\Lambda(\boldsymbol{k})$. Guided by the LPA, in section 3 a truncation

scheme of the hierarchy of flow equations for the irreducible vertices in the broken symmetry phase is introduced, which allows us to calculate the momentum-dependent self-energy $\Sigma(\boldsymbol{k})$. In section 4 we derive the two-parameter scaling function for the self-energy using different approximations. We end in section 5 with a brief summary and mention further applications of our method.

2. Exact RG flow equations in the broken symmetry phase

Our starting point is the following classical action,

$$S[\varphi] = \int \mathrm{d}^D r \left[\frac{1}{2}(\nabla\varphi)^2 + \frac{r_{\Lambda_0}}{2}\varphi^2 + \frac{u_{\Lambda_0}}{4!}\varphi^4 \right], \quad (1)$$

where an ultraviolet cutoff Λ_0 is implicitly understood. In the broken symmetry phase the Fourier transform φ_k has a finite vacuum expectation value,

$$\varphi_k = \varphi_k^0 + \Delta\varphi_k, \qquad \varphi_k^0 = (2\pi)^D \delta(\boldsymbol{k})M, \quad (2)$$

with $\langle\Delta\varphi_k\rangle = 0$. By substituting this expression into (1) and expanding in powers of $\Delta\varphi_k$, we also generate terms involving one and three powers of $\Delta\varphi_k$. Demanding that the vertex $\Gamma_{\Lambda_0}^{(1)}$ associated with the term linear in $\Delta\varphi_k$ should vanish, we obtain the magnetization in the Landau approximation,

$$M_{\Lambda_0} = \begin{cases} 0 & \text{for } r_{\Lambda_0} > 0 \\ \sqrt{-6r_{\Lambda_0}/u_{\Lambda_0}} & \text{for } r_{\Lambda_0} < 0. \end{cases} \quad (3)$$

This will serve as the initial condition for the flow equation of the order parameter in our functional RG approach.

To derive an exact hierarchy of flow equations for the vertices of our model, we introduce a momentum cutoff Λ into the free propagator separating fluctuations with small momenta $|\boldsymbol{k}| \lesssim \Lambda$ from those with large momenta $|\boldsymbol{k}| \gtrsim \Lambda$. Differentiating the generating functional Γ of the one-particle irreducible vertices with respect to Λ and expanding Γ in powers of the fields, we obtain a formally exact hierarchy of FRG flow equations for the vertices [3]. To take into account symmetry breaking, we follow the approach proposed in [14] and demand that, for all values of Λ, the flowing vertex $\Gamma_\Lambda^{(1)}$ associated with the term linear in the fluctuation $\Delta\varphi_k$ vanishes. This yields a renormalization group equation for the flowing order parameter M_Λ. We would like to calculate the true order parameter $M = \lim_{\Lambda\to 0} M_\Lambda$ and the true momentum-dependent single-particle Green function $G(\boldsymbol{k}) = \lim_{\Lambda\to 0} G_\Lambda(\boldsymbol{k})$, which we parameterize in terms of an irreducible self-energy $\Sigma(\boldsymbol{k})$,

$$G(\boldsymbol{k}) = \frac{1}{k^2 + \Sigma(\boldsymbol{k})}. \quad (4)$$

For our purpose it is convenient to include the term proportional to r_{Λ_0} in (1) in the definition of the self-energy. In the broken symmetry phase, the initial condition for the self-energy at scale $\Lambda = \Lambda_0$ is then

$$\Sigma_{\Lambda_0}(\boldsymbol{k}) = r_{\Lambda_0} + \frac{u_{\Lambda_0}}{2}M_{\Lambda_0}^2 = \frac{u_{\Lambda_0}}{3}M_{\Lambda_0}^2 = -2r_{\Lambda_0}, \quad (5)$$

where M_{Λ_0} is given in (3). As we reduce the cutoff, the evolution of the self-energy is determined by the following exact RG flow equation [14, 28],

$$\partial_\Lambda \Sigma_\Lambda(\boldsymbol{k}) = -\frac{1}{2} \int \frac{\mathrm{d}^D k'}{(2\pi)^D} \dot{G}_\Lambda(\boldsymbol{k}') \Gamma_\Lambda^{(4)}(\boldsymbol{k}', -\boldsymbol{k}', \boldsymbol{k}, -\boldsymbol{k})$$
$$- \int \frac{\mathrm{d}^D k'}{(2\pi)^D} \dot{G}_\Lambda(\boldsymbol{k}') G_\Lambda(\boldsymbol{k}' + \boldsymbol{k})$$
$$\times \Gamma_\Lambda^{(3)}(\boldsymbol{k}, -\boldsymbol{k} - \boldsymbol{k}', \boldsymbol{k}') \Gamma_\Lambda^{(3)}(-\boldsymbol{k}', \boldsymbol{k} + \boldsymbol{k}', -\boldsymbol{k})$$
$$+ (\partial_\Lambda M_\Lambda) \Gamma_\Lambda^{(3)}(\boldsymbol{k}, -\boldsymbol{k}, 0), \quad (6)$$

while the flowing order parameter M_Λ satisfies [14]

$$(\partial_\Lambda M_\Lambda) \Sigma_\Lambda(0) = -\frac{1}{2} \int \frac{\mathrm{d}^D k}{(2\pi)^D} \dot{G}_\Lambda(\boldsymbol{k}) \Gamma_\Lambda^{(3)}(\boldsymbol{k}, -\boldsymbol{k}, 0). \quad (7)$$

The flow equations for the three-point vertex $\Gamma_\Lambda^{(3)}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3)$ and the four-point vertex $\Gamma_\Lambda^{(4)}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3, \boldsymbol{k}_4)$ in the presence of symmetry breaking have been written down diagrammatically in [14]. In the present work we shall not need these equations.

In the broken symmetry phase the appropriate choice of the cutoff procedure is a delicate matter. The simplest choice is perhaps a sharp cutoff in momentum space, where the propagator is, for $|\boldsymbol{k}| < \Lambda_0$, given by [3]

$$G_\Lambda(\boldsymbol{k}) = \frac{\Theta(|\boldsymbol{k}| - \Lambda)}{\boldsymbol{k}^2 + \Sigma_\Lambda(\boldsymbol{k})}, \quad (8)$$

and the corresponding single-scale propagator is

$$\dot{G}_\Lambda(\boldsymbol{k}) = -\frac{\delta(|\boldsymbol{k}| - \Lambda)}{\Lambda^2 + \Sigma_\Lambda(\boldsymbol{k})}. \quad (9)$$

While in the symmetric phase the sharp cutoff is very convenient [3, 17], it leads to technical complications in the broken symmetry phase (see the discussion after (36) below). These can be avoided if we use a smooth cutoff procedure, which we implement via an additive regulator $R_\Lambda(\boldsymbol{k})$ in the inverse propagator [5]. The cutoff-dependent propagator is then

$$G_\Lambda(\boldsymbol{k}) = \frac{1}{\boldsymbol{k}^2 + \Sigma_\Lambda(\boldsymbol{k}) + R_\Lambda(\boldsymbol{k})}, \quad (10)$$

and corresponding single-scale propagator is

$$\dot{G}_\Lambda(\boldsymbol{k}) = [-\partial_\Lambda R_\Lambda(\boldsymbol{k})] G_\Lambda^2(\boldsymbol{k}). \quad (11)$$

At this point it is not necessary to specify the cutoff function $R_\Lambda(\boldsymbol{k})$ completely, except that we require it to be of the form[1,2]

$$R_\Lambda(\boldsymbol{k}) = (1 - \delta_{\boldsymbol{k},0}) \Lambda^2 Z_l^{-1} R(\boldsymbol{k}^2/\Lambda^2), \quad (12)$$

---

[1] As discussed in [14], it is convenient to choose the free propagator such that the product of the inverse free propagator and the order parameter field vanishes, $[G_\Lambda^0(\boldsymbol{k})]^{-1} \varphi_k^0 = 0$. The factor of $1 - \delta_{\boldsymbol{k},0}$ in (12) is introduced to satisfy this condition. Without this factor, the right-hand side of (7) would contain an additional term $\partial_\Lambda [R_\Lambda(0) M_\Lambda]$. Because the RG eliminates only modes with $\boldsymbol{k} \neq 0$, the projector $1 - \delta_{\boldsymbol{k},0}$ does not affect the RG flow at any finite $\boldsymbol{k}$, while the fluctuation of the zero mode $\Delta\varphi_{\boldsymbol{k}=0}$ can be ignored in the thermodynamic limit [14].
[2] Our factor Z_l is the inverse of the corresponding quantity Z_k introduced in [5].

where $R(x)$ is some dimensionless function satisfying $R(\infty) = 0$ and $R(0) = 1$. The inverse of the flowing wavefunction renormalization factor is given by

$$Z_l^{-1} = 1 + \left. \frac{\partial \Sigma_\Lambda(\boldsymbol{k})}{\partial \boldsymbol{k}^2} \right|_{\boldsymbol{k}^2=0}, \quad (13)$$

where $l = -\ln(\Lambda/\Lambda_0)$. The introduction of Z_l^{-1} in (12) is necessary to preserve the re-parametrization invariance of physical quantities under a rescaling of the fields [5] (see footnote 2). For explicit calculations we shall use the Litim cutoff [29],

$$R(x) = (1 - x)\Theta(1 - x). \quad (14)$$

Another popular choice is [5]

$$R(x) = \frac{x}{\mathrm{e}^x - 1}, \quad (15)$$

which has the advantage of being analytic, but leads to more complicated integrals.

3. Truncated flow equation for the momentum-dependent self-energy

The right-hand side of (6) depends on the vertices $\Gamma_\Lambda^{(3)}$ and $\Gamma_\Lambda^{(4)}$ with three and four external legs, which satisfy more complicated flow equations [14, 28] involving higher-order vertices. Keeping in line with the derivative expansion for the effective action [5], we truncate the hierarchy as follows [14]:

$$\Sigma_\Lambda(0) \approx \frac{u_\Lambda}{3} M_\Lambda^2, \quad (16a)$$

$$\Gamma_\Lambda^{(3)}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) \approx u_\Lambda M_\Lambda, \quad (16b)$$

$$\Gamma_\Lambda^{(4)}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3, \boldsymbol{k}_4) \approx u_\Lambda. \quad (16c)$$

This truncation of the field expansion is motivated by the LPA with quartic approximation for the effective potential U_{eff}, where one approximates the generating functional Γ by [5]

$$\Gamma[\varphi] \approx \int \mathrm{d}^D r \, U_{\mathrm{eff}}[\varphi^2(\boldsymbol{r})], \quad (17)$$

with

$$U_{\mathrm{eff}}[\varphi] \approx \frac{u_\Lambda}{4!} [\varphi^2 - M_\Lambda^2]^2. \quad (18)$$

The completely local character of all correlations in the LPA is known to be a surprisingly good approximation in the scaling regime close to criticality [5]. Outside the critical regime the LPA fares less well and, in general, cannot reproduce the structure known from perturbation theory (in the case considered here, only the leading order from perturbation theory will be reproduced). The LPA combined with a derivative expansion converges best if one expands around the local minimum of U_{eff}; see [30]. The condition that M_Λ is the flowing minimum leads to a flow equation for M_Λ, which in the field expansion leads to (16a)–(16c). In this truncation, the exact flow equation (7) for the order parameter reduces to

$$\partial_\Lambda M_\Lambda^2 = -3 \int \frac{\mathrm{d}^D k}{(2\pi)^D} \dot{G}_\Lambda(\boldsymbol{k}), \quad (19)$$

while our flow equation (6) for the self-energy becomes

$$\partial_\Lambda \Sigma_\Lambda(k) = \frac{u_\Lambda}{2} \int \frac{\mathrm{d}^D k'}{(2\pi)^D} \dot{G}_\Lambda(k') + \frac{u_\Lambda}{2} \partial_\Lambda M_\Lambda^2$$

$$- u_\Lambda^2 M_\Lambda^2 \int \frac{\mathrm{d}^D k'}{(2\pi)^D} \dot{G}_\Lambda(k') G_\Lambda(k'+k)$$

$$= -u_\Lambda \int \frac{\mathrm{d}^D k'}{(2\pi)^D} \dot{G}_\Lambda(k')$$

$$- u_\Lambda^2 M_\Lambda^2 \int \frac{\mathrm{d}^D k'}{(2\pi)^D} \dot{G}_\Lambda(k') G_\Lambda(k+k'). \tag{20}$$

By demanding that the flow of $\Sigma_\Lambda(0)$ is consistent with our truncation (16*a*), we obtain the flow equation for the effective interaction,

$$\partial_\Lambda u_\Lambda = -3u_\Lambda^2 \int \frac{\mathrm{d}^D k}{(2\pi)^D} \dot{G}_\Lambda(k) G_\Lambda(k). \tag{21}$$

The above equations (19)–(21) form a closed system of coupled integro-differential equations for the order parameter M_Λ, the effective interaction u_Λ, and the momentum dependent self-energy $\Sigma_\Lambda(k)$. In contrast to the LPA, these equations can be used to calculate the full k-dependence of $\Sigma(k)$. Our truncation is similar in spirit but not identical to the more elaborate truncation proposed in [18], where the LPA was also used as a guide to propose a truncation of the hierarchy of flow equations for the momentum-dependent vertices generated in the field expansion. However, unlike our equation (6), the flow equation for the self-energy proposed by Blaizot *et al* [18] does not involve the flowing order parameter, because these authors approach the critical point using an expansion around the symmetric state.

At this point it is convenient to rescale all quantities to reveal their scaling dimensions. We define dimensionless momenta $q = k/\Lambda$ and the dimensionless coupling constants

$$u_l = K_D Z_l^2 \Lambda^{D-4} u_\Lambda, \tag{22}$$

$$M_l^2 = \frac{M_\Lambda^2}{Z_l K_D \Lambda^{D-2}}, \tag{23}$$

which are considered to be functions of $l = -\ln(\Lambda/\Lambda_0)$. Here K_D is defined by

$$K_D = \frac{\Omega_D}{(2\pi)^D} = \frac{2^{1-D}}{\pi^{D/2}\Gamma(D/2)}, \tag{24}$$

where Ω_D is the surface area of the D-dimensional unit sphere. We also define the rescaled exact propagator,

$$G_l(q) = \frac{\Lambda^2}{Z_l} G_\Lambda(k)$$

$$= \frac{1}{Z_l q^2 + \Gamma_l^{(2)}(q) + R(q^2)}, \tag{25}$$

and the corresponding single-scale propagator

$$\dot{G}_l(q) = \dot{R}_l(q) G_l^2(q), \tag{26}$$

where

$$\dot{R}_l(q) = -\frac{Z_l}{\Lambda} \partial_\Lambda R_\Lambda(k) = -(2-\eta_l)R(q^2) + 2q^2 R'(q^2). \tag{27}$$

Here $R'(x) = \mathrm{d}R(x)/\mathrm{d}x$ and $\eta_l = -\partial_l \ln Z_l$ is the flowing anomalous dimension. For the Litim cutoff, $R'(x) = -\Theta(1-x)$, so that

$$\dot{R}_l(q) = [-2 + \eta_l(1-q^2)]\Theta(1-q^2). \tag{28}$$

The rescaled propagator (25) depends on the rescaled irreducible self-energy,

$$\Gamma_l^{(2)}(q) = \frac{Z_l}{\Lambda^2} \Sigma_\Lambda(k). \tag{29}$$

By construction, the constant part of the rescaled self-energy is

$$\Gamma_l^{(2)}(0) = \frac{u_l}{3} M_l^2 = \frac{Z_l}{\Lambda^2} \frac{u_\Lambda}{3} M_\Lambda^2. \tag{30}$$

The flow of $\Gamma_l^{(2)}(0)$ is thus determined by the flow of M_l^2 and u_l, which in our truncation is given by

$$\partial_l M_l^2 = (D - 2 + \eta_l)M_l^2 + 3\int_q \dot{G}_l(q), \tag{31}$$

$$\partial_l u_l = (4 - D - 2\eta_l)u_l + 3u_l^2 \int_q \dot{G}_l(q)G_l(q), \tag{32}$$

where $\int_q = \Omega_D^{-1}\int \mathrm{d}^D q$. To calculate η_l, we need the momentum-dependent part of the rescaled self-energy,

$$\gamma_l(q) = \Gamma_l^{(2)}(q) - \Gamma_l^{(2)}(0), \tag{33}$$

which satisfies

$$\partial_l \gamma_l(q) = (2 - \eta_l - q \cdot \nabla_q)\gamma_l(q) + \dot{\gamma}_l(q), \tag{34}$$

where

$$\dot{\gamma}_l(q) = u_l^2 M_l^2 \int_{q'} \dot{G}_l(q')[G_l(q'+q) - G_l(q')]. \tag{35}$$

The flowing anomalous dimension is then given by

$$\eta_l = \left.\frac{\partial \dot{\gamma}_l(q)}{\partial q^2}\right|_{q^2=0}. \tag{36}$$

The reason why using a sharp cutoff in the broken symmetry phase leads to technical complications is that, in this case, the expansion of the integral (35) for small q would start with a non-analytic term proportional to $|q|$, which requires the introduction of an additional relevant coupling constant [17]. On the other hand, with the Litim cutoff (14) or the analytic cutoff (15), the leading term in the expansion of $\dot{\gamma}_l(q)$ is proportional to q^2. In this case the expansion of the right-hand side of (35) for small q yields for the flowing anomalous dimension

$$\eta_l = -u_l^2 M_l^2 \int_q \dot{G}_l(q)\{G_l^2(q)[1 + R'(q^2)]$$

$$+ \frac{q^2}{D}[2G_l^2(q)R''(q^2) - 4G_l^3(q)[1 + R'(q^2)]^2]\}, \tag{37}$$

where $R''(x) = \mathrm{d}^2 R(x)/\mathrm{d}x^2$. For the Litim cutoff, $R''(x) = \delta(1-x)$.

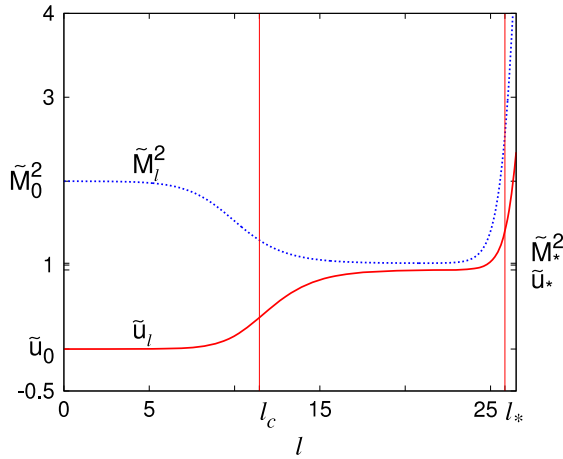**Figure 1.** Typical non-critical flow of the coupling parameters M_l^2 and u_l obtained from (41) and (42) in $D = 3$.
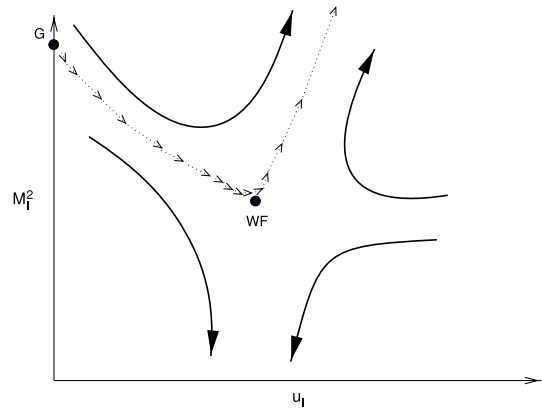

Figure 2. Qualitative flow diagram for the couplings u_l and M_l^2. The arrows sketch a nearly critical flow, with the size of the arrows representing the velocity of the flow. The dots mark the Gaussian fixed point (G) and the Wilson–Fisher fixed point (WF).

4. Two-parameter scaling in the broken symmetry phase

4.1. Truncation with only marginal and relevant couplings

In the simplest self-consistent approximation, we expand the momentum-dependent part $\gamma_l(q)$ of the two-point vertex on the right-hand side of our flow equations (31), (32), (34) to first order in q^2. Since, by definition,

$$Z_l = 1 - \left. \frac{\partial \Gamma_l^{(2)}(q)}{\partial q^2} \right|_{q^2=0} = 1 - \left. \frac{\partial \gamma_l(q)}{\partial q^2} \right|_{q^2=0}, \quad (38)$$

this amounts to approximating the propagator on the right-hand side of the flow equations (31), (32), (34) by

$$G_l(q) \approx \frac{1}{q^2 + \rho_l + R(q^2)}, \quad (39)$$

where

$$\rho_l = \Gamma_l^{(2)}(0) = \frac{u_l}{3} M_l^2, \quad (40)$$

see (30). The resulting system of flow equations for the coupling constants M_l^2 and u_l, together with the flow equation $\partial_l Z_l = -\eta_l Z_l$ for Z_l, are equivalent to the quartic approximation for the effective potential with wavefunction renormalization [5]. Using the Litim cutoff (14), equations (31) and (32) become

$$\partial_l M_l^2 = (D - 2 + \eta_l) M_l^2 - \frac{6(2 + D - \eta_l)}{D(D+2)} G_l^2(0), \quad (41)$$

$$\partial_l u_l = (4 - D - 2\eta_l) u_l - \frac{6(2 + D - \eta_l)}{D(D+2)} u_l^2 G_l^3(0), \quad (42)$$

where $G_l(0) \approx [1 + \rho_l]^{-1}$ is the rescaled propagator at zero momentum. Moreover, with the Litim cutoff the flowing anomalous dimension (37) is simply

$$\eta_l = \frac{1}{D} u_l^2 M_l^2 G_l^4(0). \quad (43)$$

Equations (41)–(43) form a closed system of differential equations for M_l^2, u_l and η_l which can easily be solved

numerically. To find the flow along the critical surface, we need to fine tune carefully the initial values u_0 and M_0^2. A typical flow of the rescaled parameters as a function of l is shown in figure 1, while in figure 2 we show the flow schematically in the (u_l, M_l^2)-plane. The Wilson–Fisher fixed point in $D = 3$ is, in this approximation, at $u_* \approx 0.942$ and $M_*^2 \approx 1.022$. As can be seen in figure 2, the couplings initially flow very slowly and stay close to their initial values in the vicinity of the Gaussian fixed point. At a characteristic scale l_c they are rapidly attracted by the Wilson–Fisher fixed point, where the flow is again almost stationary. Finally, at the scale l_*, all non-critical RG trajectories rapidly move away from the Wilson–Fisher point and the l-dependence of the couplings u_l and M_l^2 is determined by their scaling dimensions, $u_l \propto e^{\epsilon l}$ and $M_l^2 \propto e^{(D-2)l}$, where $\epsilon = 4 - D$; the flow of the unrescaled couplings u_Λ and M_Λ^2 then stops.

What determines the two characteristic scales l_c and l_*? The momentum scale $k_c = \Lambda_0 e^{-l_c}$ associated with l_c measures the size of the Ginzburg critical region. For small initial values of u_0, the logarithmic scale l_c is given by [22, 17, 19]

$$l_c \approx \frac{1}{\epsilon} \ln \left(\frac{u_*}{u_0} \right), \quad (44)$$

where u_* is the value of u_l at the Wilson–Fisher fixed point. This scale can be derived from (41) and (42) by approximating $G_l(0) \approx 1$ and $\eta_l \approx 0$. In the intermediate regime $l_c \lesssim l \lesssim l_*$ the flowing M_l^2 can then be replaced by a constant $M_l^2 \approx M_*^2 = 6/[D(D-2)]$, while for all l the solution of (42) can be approximated by

$$\frac{u_l}{u_*} \approx \frac{1}{e^{\epsilon(l-l_c)} + 1}, \quad (45)$$

where $u_* = D\epsilon/6$. The numerically obtained flow shown in figure 1 further reveals that the scale l_c is also characteristic for the l-dependence of M_l^2.

Non-critical flows which describe the system at $T < T_c$ eventually obey $M_l^2/M_*^2 \gg G_l^2(0)$ and $u_*/u_l \gg G_l^3(0)$. In that case, the solution for M_l^2 and u_l depend exponentially on

l, corresponding to trivial scaling. The unrescaled variables u_Λ and M_Λ^2 then approach finite limits, and also the physical correlation length ξ, which is defined via

$$\xi^{-2} = \lim_{\Lambda \to 0} [Z_\Lambda \Sigma_\Lambda(0)], \qquad (46)$$

remains finite. The scale l_* in figure 1 is related to ξ via $\xi^{-1} = \Lambda_0 e^{-l_*}$, or equivalently

$$2l_* = -\ln\left[\lim_{l\to\infty} e^{-2l}\Gamma_l^{(2)}(0)\right]. \qquad (47)$$

From the linearized flow around the Wilson–Fisher fixed point we obtain, in $D = 3$, the critical exponents $\nu \approx 0.553$ and $\eta \approx 0.099$. Very similar results are obtained using the analytic cutoff (15). The poor comparison of our results to the established values $\nu = 0.64$ and $\eta = 0.044$ (see [5]) can be traced to the low-order truncation of our effective potential (18); see [30, 31]. Close to $D = 4$ we obtain in the leading-order ϵ-expansion $\eta \sim \epsilon^2/12$ and $\nu \sim 1/2 + \epsilon/12$. While the result for ν is correct, the value for η compares badly with the known expansion $\eta \sim \epsilon^2/54$. This clearly shows the limitations of a low-order effective potential approximation.

4.2. FRG enhanced perturbation theory

So far, we have truncated the self-energy retaining only its marginal and relevant parts. This is a good approximation for small momenta k. On the other hand, for large k this approximation cannot correctly reproduce the momentum dependence of the self-energy which arises from perturbation theory. To leading order in the relevant dimensionless bare coupling $\bar{u}_0 = u_{\Lambda_0}\xi^{4-D}$, the perturbative momentum dependence of $\Sigma(k)$ in the broken symmetry phase is given by [20]

$$\Sigma(k) - \Sigma(0) = \xi^{-2}\Delta\sigma_0^-(k\xi), \qquad (48)$$

with

$$\Delta\sigma_0^-(x) = \frac{3\bar{u}_0}{2}[\chi(0) - \chi(x)] + O(\bar{u}_0^2), \qquad (49)$$

where

$$\chi(p) = \int \frac{d^D p'}{(2\pi)^D} \frac{1}{[p'^2 + 1][(p'+p)^2 + 1]}. \qquad (50)$$

Equations (48)–(50) are only accurate sufficiently far away from the critical point where ξ and the relevant dimensionless coupling $\bar{u}_0 = u_{\Lambda_0}\xi^{4-D}$ are small.

We now present an improved approximation for the momentum-dependent self-energy which we call *FRG enhanced perturbation theory*, since it embeds the perturbative expansion into a functional renormalization [32] such that it reproduces exactly the leading-order perturbative behavior for large k. However, in contrast to perturbation theory, FRG enhanced perturbation theory does not suffer from any divergencies; it yields an explicit description of the entire crossover to the critical regime and gives reasonable results even at the critical point.

Quite generally, the physical self-energy can be written as an integral over the entire RG trajectory [20],

$$\Sigma(k) - \Sigma(0) = \Lambda_0^2 \int_0^\infty dl\, e^{-2l + \int_0^l d\tau\, \eta_\tau} \dot{\gamma}\left(e^l k/\Lambda_0\right), \qquad (51)$$

where $\Sigma(0) = Z^{-1}\xi^{-2}$. In general, the expression for the inhomogeneity $\dot{\gamma}(q)$ will also depend on the momentum dependence of the three- and four-point irreducible vertices, as can be inferred from (6). To make progress, we employ the truncation (16a)–(16c) which leads to the approximation (35) for the inhomogeneity $\dot{\gamma}(q)$. Only the momentum-independent parts of the three- and four-point vertices enter and $\dot{\gamma}(q)$ is then completely determined by the self-energy and the order parameter alone. While this greatly simplifies the calculation of the self-energy since it leads to a closed set of equations, solving (51) remains non-trivial, since the solution for the self-energy must be determined self-consistently. The FRG enhanced perturbation theory provides for a non-self-consistent approximation to the solution of (51). In the FRG enhanced perturbation theory, the calculation of the subtracted inhomogeneity $\dot{\gamma}(q)$ via (35) is simplified by keeping only the first two terms in a momentum expansion of the self-energy. This amounts to the substitution

$$\Gamma_l^{(2)}(q) \to \Gamma_l^{(2)}(0) + (1 - Z_l)q^2 \qquad (52)$$

and the approximation (39) for the propagator on the right-hand side of (35). Within this approximation, the flows of M_l^2, u_l, and η_l are determined from (41)–(43). A similar truncation strategy has been adopted in [17] for the symmetric phase of the $O(2)$-model, and in [33] to calculate the spectral function of the Tomonaga–Luttinger model. A comparison with the completely self-consistently determined self-energy is presented at the end of this section, where we show that the error arising from the non-self-consistency of the FRG solution is extremely small. Perturbation theory is recovered when the flow of the running couplings is approximated by their trivial l-dependence arising from their scaling dimensions. The self-energy can now be expressed in terms of a two-parameter scaling function,

$$\Sigma(k) = k_c^2 \sigma^-(x, y), \qquad (53)$$

with $x = k\xi$ and $y = k/k_c$. The ratio of these variables is then $x/y = e^{l_* - l_c}$. If we introduce

$$\Delta\sigma^-(x, y) = \sigma^-(x, y) - k_c^2\Sigma(0) = \sigma^-(x, y) - y^2/Zx^2, \qquad (54)$$

this leads to

$$\Delta\sigma^-(x, y) = \int_0^\infty dl\, e^{-2(l - l_c) + \int_0^l d\tau\, \eta_\tau} \dot{\gamma}_l(e^{l - l_c} y)$$
$$= y^2 \int_{ye^{-l_c}}^\infty dp\, p^{-3} Z_{l_c + \ln(p/y)}^{-1} \dot{\gamma}_{l_c + \ln(p/y)}(p), \qquad (55)$$

where we substituted $p = ye^{l - l_c}$ and used $Z_l = e^{-\int_0^l d\tau\, \eta_\tau}$. The asymptotic behavior of $\dot{\gamma}_l(q)$ for small q is $\dot{\gamma}_l(q) \approx \eta_l q^2$. For large q it approaches a constant which, using the Litim cutoff, is

$$\lim_{q\to\infty} \dot{\gamma}_l(q) \approx 2u_l^2 M_l^2 G_l^3(0) \frac{(2 + D - \eta_l)}{D(D+2)}. \qquad (56)$$
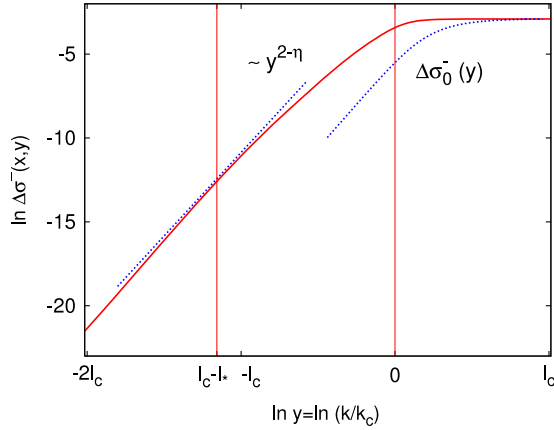
Figure 3. Typical behavior of the two-parameter scaling function $\Delta\sigma^-(x, y)$ defined in (55) for $x = e^{l_* - l_c} y$. The initial coupling parameters are $u_0 = 0.005$ and $M_0^2 = 1.948\,2092$, which yields $l_c \simeq 5.23$ and $l_* \simeq 11.29$.

Figure 4. Comparison of two different approximations of the self-energy at criticality, as discussed in the text: self-consistent numerical solution $\sigma_{\mathrm{nu}}^-(\infty, k/k_c)$ of (19)–(21) and solution $\sigma^-(\infty, k/k_c)$ based on the substitution (52) on the right-hand sides of these flow equations.

In $D = 3$ the function $\dot{\gamma}_l(q)$ can be calculated analytically for the Litim cutoff; the result is given in the appendix. At the critical point, $x \to \infty$ since the correlation length diverges, so the scaling function reduces to

$$\Delta\sigma^-(\infty, y) = \sigma^-(\infty, y) = \sigma_*(y). \qquad (57)$$

The asymptotic behavior of the scaling function for both very small and very large y follows directly from (55). For $y \ll 1$, i.e. in the critical long wavelength regime [20], the lower limit of integration may be replaced by zero and all coupling parameters may be replaced by their fixed point values. Then we find

$$\sigma_*(y) \approx A_D y^{2-\eta}, \qquad (58)$$

where η is the fixed point value of η_l and

$$A_D = \int_0^\infty \mathrm{d}p \; p^{\eta-3} \dot{\gamma}_*(p), \qquad (59)$$

with $\gamma_*(p) = \lim_{l \to \infty} \gamma_l(p)$. In $D = 3$ we obtain numerically $A_3 \approx 1.075$. In the critical long wavelength regime $y \gg 1$, one may approximate all couplings by their initial values. The scaling function then approaches the constant value

$$\sigma_*(y) \approx \frac{2}{D} u_0^2 M_0^2 G_0^3(0). \qquad (60)$$

Such a constant plateau is expected from the structure of the truncation employed. In fact, for $k \gg k_c$, one expects that the momentum dependence of the self-energy is that of lowest-order perturbation theory; see the discussion at the beginning of this subsection. However, an effective ultraviolet cutoff is now provided by k_c^{-1} which regularizes the theory in place of the correlation length ξ, which is infinite at criticality. While this is indeed the leading-order correction to the self-energy in an expansion in powers of the bare interaction strength, the correct form of the self-energy should further display a $\ln(k/k_c)$ dependence at large k with a pre-factor which is quadratic in the bare interaction [17, 20]. The reason for the absence of such a term in the present approximation is that
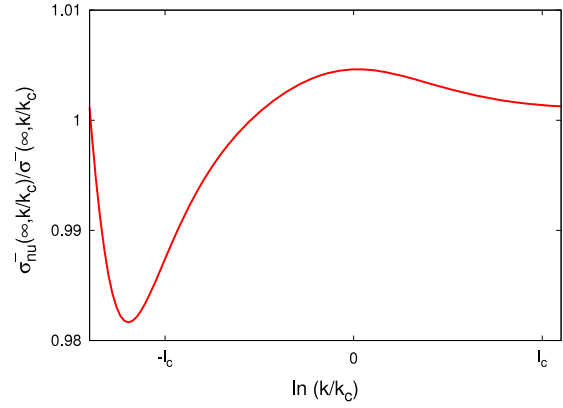
our truncation for the four-point vertex in (16c) does not take vertex corrections into account. In the macroscopically ordered regime $k\xi \ll 1$, we find that $\Sigma(k) - \Sigma(0)$ vanishes as k^2, as can be seen in figure 3.

For the present model, it is of course possible to calculate the solution of the coupled integro-differential equations (19)–(21) exactly without any further approximations. One obtains completely self-consistent solutions for the flow of the self-energy $\Sigma_\Lambda(k)$ and the order parameter if the truncation of the momentum dependence of Σ_Λ and G_Λ on the right-hand sides of (19)–(21) using the substitution (52) is omitted. A comparison of the completely self-consistent solution $\sigma_{\mathrm{nu}}^-(\infty, y)$ for the scaling function with the scaling function $\sigma^-(\infty, y)$ obtained within the FRG enhanced perturbation theory, i. e. with the help of the substitution (52) on the right-hand sides of (19)–(21), is shown in figure 4. Obviously, the relative error due to the substitution (52) is remarkably small, so we conclude that our substitution (52) is quite accurate.

Of course, one could easily improve the approximations presented here. A straightforward extension would be to truncate the effective potential U_{eff} in (17) at some higher order $n > 2$,

$$U_{\mathrm{eff}}[\varphi^2] \approx \sum_{m=2}^n \frac{u_\Lambda^{(m)}}{(2m)!} \left[\varphi^2 - M_\Lambda^2\right]^m. \qquad (61)$$

To arrive at the flow equations of the parameters $u_\Lambda^{(2)}, \ldots, u_\Lambda^{(n)}$, one would need to take into account the flow equation of the lowest n vertices of a field expansion. We have done so up to $n = 5$; a significant improvement of the critical exponents η and ν in the Ising model would, however, require approximately $n = 10$, as is known from previous investigations of the derivative expansion [31]. Note that our FRG enhanced perturbation theory can also be adopted if arbitrarily higher orders in n are included. This is because, within our approach, the flow of the coupling constants that parameterize the local potential depends only on the lowest-order momentum expansion of the self-energy and can be calculated exactly as in the usual derivative expansion [31].

Once the flow of the local potential is known, the higher-order momentum dependence of the self-energy can be determined. Of course, not all models have a structure as simple as the one discussed here, which permits us to include arbitrarily high orders of the local potential. One may wonder whether our approach is also useful for describing more complicated systems. It is certainly expected to be useful in non-critical interacting systems, such as interacting bosons in two or three dimensions, where a low-order truncation of the effective action should suffice [34]. An accurate description of the momentum-dependent self-energy of more complicated and possibly critical systems, such as frustrated spin models [6], is a very challenging task which we have not yet attempted. While including all orders of the local potentials would then be prohibitive, a low-order truncation might yet be qualitatively correct.

5. Summary and conclusions

Let us summarize the three main results of this work:

(i) We have demonstrated how symmetry breaking can be included in the exact hierarchy of FRG flow equations for the irreducible vertices within the framework of the field expansion, using the approach from [14]. The basic idea is to require that the vertex with one external leg vanishes identically for all values of the flow parameter, which yields an additional flow equation for the order parameter. Our method differs from those employed in [11–13], which do not explicitly include a flow equation for the order parameter.

(ii) Guided by the LPA, we have proposed a simple truncation of the FRG flow equation for the momentum-dependent self-energy $\Sigma(k)$, which yields a reasonable interpolation between the perturbative regime for large momenta and the critical regime for $k \to 0$. We have also pointed out that a sharp Θ-function cutoff (which is still very popular in the condensed matter community) is not suitable for analyzing the broken symmetry phase, because it generates a non-analytic term proportional to $|k|$ in the two-point vertex for momentum scales below the RG cutoff Λ.

(iii) Using the above truncation, we have calculated the two-parameter scaling function $\sigma^-(k\xi, k/k_c)$ describing the scaling of the self-energy in the broken symmetry phase slightly below the critical temperature. Similarly to the case $T > T_c$ discussed in [20], the scaling function depends on two parameters, involving the correlation length ξ and the Ginzburg scale k_c. The latter remains finite at the critical point and measures the size of the critical region.

It is straightforward to generalize the method described here to study spontaneous symmetry breaking in quantum mechanical many-body systems. For example, our approach can be used to obtain fluctuation corrections to the BCS gap equation in the attractive Fermi gas [16]. Note that, beyond the BCS approximation, the superconducting order

parameter should be distinguished from the off-diagonal self-energy associated with the single-particle Green function, so it is important to introduce both quantities into the FRG as independent parameters. We are currently using our method to study the interacting Bose gas [34], where a truncation similar to the one discussed here yields corrections to the Bogoliubov mean-field approximation for the diagonal and off-diagonal self-energies which are consistent with the Hugenholtz–Pines theorem [35].

Appendix. Analytical form of $\dot{\gamma}_l(q)$ in three dimensions

Using the Litim cutoff [29] given in (14) we can rewrite the function $\dot{\gamma}_l(q)$ defined in (35) as

$$\dot{\gamma}_l(q) = u_l^2 M_l^2 G_l^3(0) \int_{q'} \dot{R}_l(q')$$

$$\times \Theta\left(|q' + q|^2 - 1\right) \frac{1 - |q' + q|^2}{|q' + q|^2 + \rho_l}, \qquad (A.1)$$

where $\dot{R}(q)$ is defined in (28). In $D = 3$ the integration in (A.1) can be performed exactly. We find

$$\dot{\gamma}_l(q) = \Theta(2 - q)\dot{\gamma}_l^<(q) + \Theta(q - 2)\dot{\gamma}_l^>(q), \qquad (A.2)$$

where the functions $\dot{\gamma}_l^<(q)$ and $\dot{\gamma}_l^>(q)$ are defined by

$$\dot{\gamma}_l^<(q) = M_l^2 u_l^2 G_l^3(0)\left\{ A_l(q) + B_l(q) \ln\left[\frac{\rho_l + (1 + q)^2}{\rho_l + 1}\right]\right.$$

$$\left. - C_l(q)\left[\arctan\left(\frac{1}{\sqrt{\rho_l}}\right) - \arctan\left(\frac{1 + q}{\sqrt{\rho_l}}\right)\right]\right\}, \qquad (A.3)$$

$$\dot{\gamma}_l^>(q) = M_l^2 u_l^2 G_l^3(0)\left\{ D_l(q) + E_l(q) \ln\left[\frac{\rho_l + (1 + q)^2}{\rho_l + (1 - q)^2}\right]\right.$$

$$\left. + C_l(q)\left[\arctan\left(\frac{1 - q}{\sqrt{\rho_l}}\right) + \arctan\left(\frac{1 + q}{\sqrt{\rho_l}}\right)\right]\right\}, \qquad (A.4)$$

with

$$A_l(q) = \frac{1}{480}\{60[1 + \rho_l][4 - \eta_l(1 + \rho_l)]$$

$$- 30q[4 - \eta_l + 4\rho_l(3 - 2\eta_l) - 7\eta_l\rho_l^2] + 20\eta_l q^2$$

$$\times [5 + 9\rho_l] - 5q^3[4 + \eta_l(17 + 25\rho_l)] - 2\eta_l q^5\}, \qquad (A.5)$$

$$B_l(q) = \frac{1 + \rho_l}{16q}\{4[q^2 - \rho_l - 1] + \eta_l$$

$$\times [\rho_l^2 + 2\rho_l(1 - 3q^2) + (q^2 - 1)^2]\}, \qquad (A.6)$$

$$C_l(q) = \frac{\sqrt{\rho_l}}{2}(1 + \rho_l)[2 - \eta_l(1 + \rho_l - q^2)], \qquad (A.7)$$

$$D_l(q) = \frac{\eta_l}{5} - \frac{1}{12}[\eta_l(5 + \rho_l) - 8]$$

$$- (1 + \rho_l)[1 - \frac{\eta_l}{4}(2 + 3\rho_l - q^2)], \qquad (A.8)$$

$$E_l(q) = \frac{1 + \rho_l}{16q}\{\eta_l - 4 + \rho_l[\eta_l(2 + \rho_l) - 4]$$

$$- 2q^2[3\eta_l\rho_l + \eta_l - 2] + \eta_l q^4\}. \qquad (A.9)$$

References

[1] Wegner F J and Houghton A 1973 *Phys. Rev.* A **8** 401
[2] Wetterich C 1993 *Phys. Lett.* B **301** 90
[3] Morris T R 1994 *Int. J. Mod. Phys.* A **9** 2411
[4] Bagnuls C and Bervillier C 2001 *Phys. Rep.* **348** 91
    Bagnuls C and Bervillier C 2002 *Phys. Rev.* E **65** 066132
[5] Berges J, Tetradis N and Wetterich C 2002 *Phys. Rep.* **363** 223
[6] Delamotte B, Mouhanna D and Tissier M 2004 *Phys. Rev.* B **69** 134413
    Delamotte B 2007 *Preprint* cond-mat/0702365
[7] Pawlowski J M 2005 *Preprint* hep-th/0512261
[8] Birse M C, Krippa B, McGovern J A and Walet N R 2005 *Phys. Lett.* B **605** 287
    Diehl S and Wetterich C 2007 *Nucl. Phys.* B **770** 206
    Diehl S, Gies H, Pawlowski J M and Wetterich C 2007 *Phys. Rev.* A **76** 021608(R)
    Diehl S, Gies H, Pawlowski J M and Wetterich C 2007 *Phys. Rev.* A **76** 053627
[9] Kopietz P and Busche T 2001 *Phys. Rev.* B **64** 155101
[10] Salmhofer M and Honerkamp C 2001 *Prog. Theor. Phys.* **105** 1
[11] Salmhofer M, Honerkamp C, Metzner W and Lauscher O 2004 *Prog. Theor. Phys.* **112** 943
[12] Honerkamp C and Salmhofer M 2005 *Prog. Theor. Phys.* **113** 1145
[13] Gersch R, Honerkamp C, Rohe D and Metzner W 2005 *Eur. Phys. J.* B **48** 349
[14] Schütz F and Kopietz P 2006 *J. Phys. A: Math. Gen.* **39** 8205
[15] Ellwanger U and Wetterich C 1994 *Nucl. Phys.* B **423** 137
[16] Lerch N, Bartosch L and Kopietz P 2008 *Phys. Rev. Lett.* at press (*Preprint* cond-mat/0710.1801)
[17] Ledowski S, Hasselmann N and Kopietz P 2004 *Phys. Rev.* A **69** 061601(R)
    Hasselmann N, Ledowski S and Kopietz P 2004 *Phys. Rev.* A **70** 063621
[18] Blaizot J-P, Méndez-Galain M and Wschebor N 2006 *Phys. Rev.* E **74** 051116
    Blaizot J-P, Méndez-Galain M and Wschebor N 2006 *Phys. Rev.* E **74** 051117
    Blaizot J-P, Méndez-Galain M and Wschebor N 2006 *Phys. Lett.* B **632** 571
    Blaizot J-P, Méndez-Galain M and Wschebor N 2007 *Eur. Phys. J.* B **58** 297
    Guerra D, Méndez-Galain R and Wschebor N 2007 *Eur. Phys. J.* B **59** 357–65
[19] Amit D J 1974 *J. Phys. C: Solid State Phys.* **7** 3369
[20] Hasselmann N, Sinner A and Kopietz P 2007 *Phys. Rev.* E **76** 040101(R)
[21] Anisimov M A, Povodyrev A A, Kulikov V D and Sengers J V 1995 *Phys. Rev. Lett.* **75** 3146
[22] Pelissetto A and Vicari E 2002 *Phys. Rep.* **368** 549
[23] Halperin B I and Hohenberg P C 1968 *Phys. Rev.* **177** 952
[24] Ma S K 1973 *Rev. Mod. Phys.* **45** 589
[25] Fisher M E 1974 *Rev. Mod. Phys.* **46** 597
[26] Baym G, Blaizot J P, Holzmann M, Laloe F and Vautherin D 1999 *Phys. Rev. Lett.* **83** 1703
[27] Baym G, Blaizot J-P and Zinn-Justin J 2000 *Europhys. Lett.* **49** 150
[28] Schütz F, Bartosch L and Kopietz P 2005 *Phys. Rev.* B **72** 035107
[29] Litim D F 2001 *Phys. Rev.* D **64** 105007
[30] Aoki K I, Morikawa K, Souma W, Sumi J L and Terao H 1998 *Prog. Theor. Phys.* **99** 451
[31] Canet L, Delamotte B, Mouhanna D and Vidal J 2003 *Phys. Rev.* D **67** 065004
[32] Wetterich C 1996 *Mod. Phys. Lett.* A **14** 2573
[33] Busche T, Bartosch L and Kopietz P 2002 *J. Phys.: Condens. Matter* **14** 8513
[34] Sinner A, Hasselmann N and Kopietz P, unpublished
[35] See, for example, Nozières P and Pines D 1990 *The Theory of Quantum Liquids* vol 2 (Redwood City: Addison-Wesley)